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Fibonacci Sequence and Anagram Timbre: Exploring the Golden Ratios on Music Composition

Hery Budiawan
Art Education Study Program, Universitas Negeri Jakarta, Jakarta, Indonesia.
Rizky Fauzy Ananda
Art Education, Universitas Pendidikan Indonesia, Indonesia
Iwan Sugihartono
Phycis Study Program, Universitas Negeri Jakarta, Indonesia
Djohan
Graduate School, Indonesia Institute of the Arts, Yogyakarta, Indonesia.
Phakkharawat Sittiprapaporn
Neuropsychological Research Laboratory, Department of Anti-Aging and Regenerative Science, School of Anti-Aging and Regenerative Medicine, Mae Fah Luang University, Bangkok, Thailand
Tesaannisa
English Literature Study Program, Universitas Negeri Jakarta, Indonesia.
R.M. Aditya Andriyanto
Music Education Study Program, Universitas Negeri Jakarta, Jakarta, Indonesia

Abstract

This research aims to explore an innovative approach to incorporating the golden ratio in musical composition by applying the Fibonacci sequence. The interplay between music and mathematics, focusing on the golden ratio's influence on timbre, is investigated. The Fibonacci sequence, renowned for its aesthetic appeal, has captivated scholars and artists for centuries. Composers, in their pursuit of harmonious compositions, utilize the golden ratio (φ) as a unique mathematical ratio believed to be inherently pleasing to the human eye. By applying the principles of the golden ratio, particularly to timbre, a fresh perspective emerges on how mathematical concepts shape artistic expression. Timbre, which is the distinctive quality of sound, is explored through the Fibonacci sequence, allowing composers to experiment with innovative approaches to structure musical elements and create captivating auditory experiences. The article also delves into the practice-led research approach employed in music composition, utilizing Artistic Research methodology that integrates mathematical theory and musicology. The results of this creative research reveal the golden ratio's influence on the tonal qualities of instruments or voices, enhancing sound synthesis parameters and adding balance and melody coherence to compositions. The Fibonacci sequence and the golden ratio thus serve as rich sources of inspiration for composers, fostering exploration in music composition, particularly regarding timbre. Integrating these mathematical principles into the artistic process allows composers to enchant audiences with intricate and aesthetically pleasing musical pieces. The ongoing synergy between mathematics and music remains a fertile ground for exploring artistic ideas and fostering innovation.

Key Words: Fibonacci, Music Composition, Mathematics, Timbre, practice-led research.

1. Introduction

Music and mathematics share a long-standing relationship, and one of their most fascinating intersections is the Fibonacci sequence. The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, and so on. (Geambaşu et al., 2020; Hutchison & Hutchison, 2010). The relationship between mathematics and music has long fascinated and inspired composers throughout history, leading them to draw inspiration from various mathematical principles to create harmonious and aesthetically pleasing compositions (Ji et al., 2020). One intriguing connection between the two fields is the relationship between the Fibonacci sequence and music composition, particularly focusing on how the golden ratio influences the timbre of musical compositions. The Fibonacci sequence, known for its inherent aesthetic appeal, has captivated mathematicians, scientists, and artists for centuries (Artincontext, 2022; Aurues, 2013). Aesthetic education within the realm of art education relies on cultivating an appreciation for developing sensitivity to both natural beauty and the beauty manifested in artistic creations, whether they be natural or artificial (Walidaini et al., 2022). Composers, driven by the desire to evoke emotions and create captivating auditory experiences, have continually sought innovative approaches to structure their musical elements.

The Fibonacci sequence, a series of numbers derived by adding the two preceding ones, has captivated diverse disciplines due to its intriguing patterns. In music, composers recognize its potential for creating complex rhythmic patterns, melodic progressions, and harmonies. The golden ratio, denoted by the Greek letter φ (phi) (Sneha Ambwani, Arup Kumar Misra, 2017; Stipancic-Klaic & Matotek, 2010) and approximately equal to 1.618, is a mathematical ratio with aesthetic qualities, revered by artists and architects (Saraf & Saraf, 2013), and involves dividing a line to achieve specific ratios. The golden ratio's influence extends to music composition, where its application is believed to contribute to visual harmony. Timbre, the distinctive quality of sound, is crucial in music composition, allowing composers to differentiate instruments and evoke emotions. This research investigates the impact of the Fibonacci sequence and the golden ratio (Fibonacci numbers and the golden ratio, 2016) on musical timbre, exploring innovative approaches to shaping sonic qualities. Applying the golden ratio to timbre in music composition, as proposed by (Mclachlan, 2016), offers a fresh perspective on how mathematical concepts shape the artistic expression of Fibonacci numbers and the golden ratio, (Budiawan & Martyastiadi, 2020; McAdams, 2013; Mielke & Andrews, 2022; Yan & Tu, 2022).

This exploration opens new possibilities for composers to create captivating auditory experiences that engage listeners on a profound level (Hagman, 2005; Smith & Dean, 2009) By incorporating these mathematical concepts into their creative process, composers can enhance the aesthetic appeal and harmonic complexity of their musical pieces. Through an analysis of selected musical compositions and experimental manipulation of sound synthesis parameters, this study seeks to uncover the intricate interplay between the Fibonacci sequence, the golden ratio, and the timbre of musical compositions. By unraveling these connections, composers can unlock a rich source of inspiration and explore new avenues in music composition.

This research embarks on a journey to explore the intriguing relationship between the Fibonacci sequence and music composition, with a specific focus on the influence of the golden ratio on timbre. By integrating mathematical concepts into the creative process, composers can create harmonious and captivating musical pieces that resonate with listeners. The interplay between mathematics and music continues to offer fertile ground for artistic exploration and innovation, and the exploration of the Fibonacci sequence and the golden ratio in music composition promises exciting possibilities for composers to push the boundaries of their creative expression. So far, musical composition has always referred to Western music as a good musical parameter, relying on which have standardizations, even though in music composition, composers can determine as freely as possible the sound they choose. This research is also a form of resistance to the "colonization" of Western music, which is still felt in the world of music composition.

The Fibonacci sequence and the golden ratio have been extensively studied and appreciated by mathematicians, scientists, and artists. The aesthetic appeal of the Fibonacci sequence has intrigued scholars for centuries. Composers, on the other hand, have long sought methods to create harmonious and pleasing compositions. The golden ratio approximated as 1.618, is believed to possess inherent beauty and harmony. Its application can be seen in various artistic domains, such as architecture, visual arts, and design. Timbre, the unique sonic quality of an instrument or voice, is a crucial element in music composition. By exploring the relationship between the Fibonacci sequence and timbre, composers can experiment with innovative approaches to structure musical elements and create captivating auditory experiences. The study of Fibonacci and anagram timbre is a concept to find composition techniques in finding new frequencies that do not depart from the tonality of Western music;

therefore, this research is a new gap in the field of music, especially in making music composition techniques in search of new frequencies.

Table 1. Fibonacci number in the sequence

n	1	2	3	4	5	6	7	8	9	10	11	12	13
Fn	1	1	2	3	5	8	13	21	34	55	89	144	233

The divisibility patterns within the Fibonacci sequence reveal interesting relationships between the numbers. For example, every third Fibonacci number denoted as F_3 is divisible by two. Similarly, every fourth Fibonacci number, F_4, is divisible by three. Continuing this pattern, every fifth Fibonacci number, F_5, is divisible by five and every sixth Fibonacci number, F_6, is divisible by eight, and so on. In general, it can be observed that every nth Fibonacci number, F_{mn}, is divisible by the nth number in the Fibonacci sequence, denoted as F_n (Browning, 2018). Fibonacci numbers in composite-number positions are always composite numbers, except for the fourth Fibonacci number. In other words, if n is not prime, the nth Fibonacci number will not be a prime (Hartshorn, 2009).

Table 2. Example Fibonacci sequence number.

F_4	=	3
F_8	=	21
F_{11}	=	89

To facilitate the process of working with anagrams and permutation procedures, this system helps in understanding the changes that occur during anagram operations. To explain, the number 123 is deliberately used as an example, but the concept can be applied to nontonal musical systems such as slendro (Purnomo, 2022), pelog (Rahn, 1978), and madenda (Fausta, 2019). The basic steps involved in this process are as follows: In Step 1, the numbers remain unchanged (123 to 123). In Step 2, the first number becomes the last number, while the second and third numbers remain in their positions (123 to 231). Step 3 involves the transformation of the third number into the first position, the first number into the second position, and the second number into the third position (123 to 312). Step 4 entails rotating the positions of the second and third numbers while keeping the first number fixed (123 to 132). In Step 5, the first series becomes the third series, the second series becomes the first series, and the third series becomes the second series (123 to 321).

The chromatic frequency of Western music is typically represented by a set of specific values (Ostendorf, 2009), such as 440 Hz, 466 Hz, 493 Hz, and so on (Katsanevaki, 2011; Wu, 2016). To obtain the next note or frequency, the subsequent series is multiplied by a factor of 2, which corresponds to moving one octave higher, or in the case of the second series, one octave lower. Applying the anagram technique from Hery Budiawan, a total of 60 new pitches can be generated by playing with the frequency of A = 440.

Table 3. Pitch with anagram technique (Budiawan, 2021)

Tone	A	Bb	B	C	C#	D	D#	E	F	F#	G	Ab
Frequency in (Hz)	440	466	493	523	554	587	622	659	698	739	783	830
1 Permutation	404	664	934	235	545	875	226	596	986	397	837	308
2 Permutation	044	646	349	352	455	758	262	965	869	937	378	083
3 Permutation	404	466	439	532	545	578	622	695	689	793	738	803
4 Permutation	044	664	394	325	455	785	226	956	896	937	387	038
5 Permutation	440	646	943	253	545	857	262	569	968	379	873	380

However, out of these 60 pitches, 12 of them are duplicates, resulting in 48 unique techniques derived from the permutation results. When sorted in an ascending order, the frequencies range from 038 Hz to 986 Hz (Budiawan, 2021). By departing from the tonal frequencies of Western music, researchers have obtained a wide range of frequency regions that serve as a foundation for their compositions. These regions span from low to high frequencies, providing ample possibilities for creative exploration in composition.

Table 4. Fibonacci Sequence and Pitch Anagram Timbre.

Fn: Fibonacci number sequence	**1**	**1**	**2**	**3**	**5**	**8**	**13**	**21**	**34**	**55**	**89**	**144**	**233**	**etc.**
Pitch Anagram Timbre (Hz)	38	44	83	226	235	253	262	308	325	349	352	378	394	etc.

In previous studies, Fibonacci was used as a tool to process melodies in musical compositions, but in this study researchers used Fibonacci and the concept of anagrams as a search for new frequencies in forming compositions that researchers had never done before

2. Methods

This study utilizes the Artistic Research methodology with a melody-based (practice-led research) approach, combining mathematical theory and musicology in the field of music composition. Artistic Research is a scientific technique that examines the art creation process from an artistic perspective, incorporating historical research to revive earlier forms of expression (Schwarz, 2009) Practice-led research is a research method that integrates creative practice, techniques, and outcomes as part of the research design and results (Candy, 2011) Creative practice necessitates creative thinking, which includes generating innovative and adaptable ideas, solutions, or insights (Budiawan & Martyastiadi, 2020; Runco & Chand, 1995). The diagram in Figure 1 illustrates the creative thinking process and its essential elements.

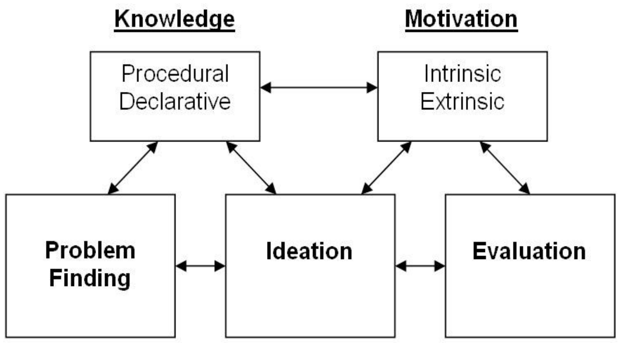

Figure 1. The creative process (Runco & Chand, 1995)

The diagram portrays two levels where the first level (Problem discovery, Ideation, and Evaluation) comprises essential abilities (skills) that are crucial to comprehend creative thinking. Meanwhile, the second level (Knowledge and Motivation) is a factor that affects but does not regulate other factors and depends on the primary factor (Runco & Chand, 1995). The stages of creation in practice-led research are flexible, and there are no fixed or structured steps, as they adapt to each researcher's distinct approach (Cunningham et al., 2017a). The following stages of creation research are commonly experienced by researchers, which are based on literature reviews and their artistic background: Data Collection (Fibonacci Sequence, Music Composition, and Anagram Timbre), Creation Deepening, and Studio Work.

3. Results

From the above data, the creation of this musical piece can be done by considering several factors, including the Fibonacci sequence and timbre anagram as data and calculations to generate a new composition. All of the mentioned elements will be accomplished through the implementation of the golden ratio of Fibonacci, which is the number 1,618 used as a divisor to create melodies and rhythms in this composition. Before delving into the diagram below, it should be noted that it represents the processed data combining the Fibonacci sequence, timbre anagram, and the golden ratio. For this research, only 13 sequences will be discussed, as the principle of the sequence will remain the same. In this study, when dividing the Fibonacci sequence by 1,618, the resulting values will be used as notes to create melodies in the musical composition, while the timbre anagram will be used as a new pitch, and the timbre anagram itself will not be divided by 1,618 to fulfill the musical harmony requirements.

Table 4. Note value with the golden ratio technique.

F_n: Fibonacci number sequence	Golden Ratio as a divisor number n = Fn/1,618
1	0,618
1	0,618
2	1,236
3	1,854
5	3,090
8	4,944
13	8,035
21	12,979
34	21,014
55	33,993
89	55,006
144	88,999
233	144,005

The division by the golden ratio of Fibonacci yields numbers that can be used as both note values and tempos in musical compositions. For instance, if $F_6 = 8$, the calculation in the sequence of 8 reveals the number 4.944, which is rounded to 5. Similarly, for $F_6 = 5$, the calculation in the musical composition results in a value of 0.75, which can be equated to ¾ in terms of note values. Rounding operations are applied in this concept due to the challenges of playing music with unconventional note values. For instance, a value such as 0.6 would be rounded or approximated to 0.75 in the context of music. Furthermore, the search for new pitches is conducted using the data source of timbre anagrams, which leads to the creation of fresh pitches in forming musical melodies.

Table 5. Pitch anagram timbre value with the golden ratio technique.

Pitch Anagram Timbre (Hz)	Golden Ratio as a divisor number n = Hz/1,618
38	23.49
44	27.19
83	51.30
226	139.68
235	145.24
253	156.37
262	161.93
308	190.36
325	200.87
349	215.70
352	217.55
378	233.62
394	243.51

The process of dividing the timbre anagram pitches by the golden ratio of 1.618 yields the generation of new pitches, as presented in the table above, which will be incorporated into the musical composition. This utilization of the golden ratio brings about an intriguing aspect, as it has the potential to produce something remarkably unique in terms of sound and compositional approach. To illustrate, let us consider an example where a timbre anagram pitch of 253 Hz is subjected to division by the golden ratio, resulting in a new pitch of 156.37 Hz. In the table, it is evident that the difference between the original timbre anagram pitch and the corresponding new pitch ranges from 14.51 Hz to 150 Hz. The emergence of these new pitches opens up exciting possibilities for musical composition. The introduction of different frequencies and tones into a composition can add depth and richness to the overall musical experience. Furthermore, the varying differences between the timbre anagram pitches and the new pitches provide a range of options for the composer to explore and manipulate. This process not only highlights the creative potential of the golden ratio but also highlights the intricate relationship between mathematical principles and artistic expression in the realm of music composition. By incorporating these newly generated pitches, composers can delve into uncharted territory and create compositions that captivate the listener with their distinctiveness and harmonious qualities.

Table 6. Music composition technique with the golden ratio value

n	**Anagram timbre (pitch) Hz**		**Note Value**	
	Original	with golden ratio Hz/1.618	with golden ratio	Rounding/approach to note values (bpm)
1	38	23.49	0.618	0.75 (3/4)
2	44	27.19	0.618	0.75 (3/4)
3	83	51.30	1.236	1,5
4	226	139.68	1.854	2
5	235	145.24	3.09	3
6	253	156.37	4.944	5
7	262	161.93	8.035	8
8	308	190.36	12.979	12 or 6
9	325	200.87	21.014	21 or 7
10	349	215.70	33.993	34 or 8
11	352	217.55	55.006	55 or 5
12	378	233.62	88.999	99 or 3
13	394	243.51	144.005	144 or 12

Table 6 represents the final results obtained from the application of the golden ratio concept as the divisor for the 13 Fibonacci sequences with timbre anagrams. Based on the data presented above, the researcher is now able to create a musical composition by utilizing the predetermined calculations derived from the Fibonacci sequence and timbre anagrams. In this article, the researcher, who is also the composer, will execute these calculated results into a musical piece. The following music composition serves as an example of the application of the Fibonacci golden ratio. It is surprising and offers a new listening experience. The composition was processed using computer technology because certain pitches are impractical to play with conventional instruments. This limitation provides a unique opportunity for the researcher to explore various possibilities in the creation of musical compositions. By embracing the principles of the golden ratio and incorporating them into musical composition, new avenues of artistic expression are discovered. Using the Fibonacci sequence and timbre anagrams not only adds complexity and depth to the composition but also offers a novel perspective on the interplay between mathematics and music. The use of computer-based tools allows for the realization of pitches that may be unattainable using traditional instruments, providing the composer with the freedom to explore uncharted sonic territories. This creative process exemplifies the researcher’s dedication to pushing the boundaries of musical experimentation and highlights the innovation potential when mathematical concepts are integrated into artistic endeavors. The resulting musical compositions not only captivate the listener with their unconventional arrangements but also provide a platform for further exploration and discovery within the realm of creating music.

Music composition with the golden ratio

In this work, the Fibonacci sequence and the golden ratio are applied to create a unique and intriguing musical structure. The Fibonacci sequence is a series of numbers where each number is the sum of the two preceding numbers (e.g., 1, 1, 2, 3, 5, 8, and so on). The golden ratio (phi, φ), is a mathematical proportion often found in nature and art. In this context, the golden ratio is used to determine the proportions and relationships between musical elements such as duration, pitch, and specific notes. In this composition, only a portion of the permutation of pitch anagram timbres is selected for use. This may be due to the complexity and variation of pitch that can be generated from these permutations. However, by using only a portion of these permutations, the composer can create a significant variation within the musical structure. The duration of the piece, reaching 240 s, demonstrates careful planning and arrangement. However, with further development, this work has the potential to become a longer composition, reaching 240 minutes. In the measurement of musical time, each beat is counted as 4 counts, which may indicate complex and diverse rhythmic patterns in this work.

This composition highlights the creative application of the Fibonacci sequence and the golden ratio in creating a unique musical composition. The use of a partial permutation of pitch anagram timbres and specific time measurements gives this work its distinctive character. The potential for expanding the duration of the piece and further exploration of the combination of the Fibonacci sequence, golden ratio, and other musical elements provide room for further creative exploration in music composition.

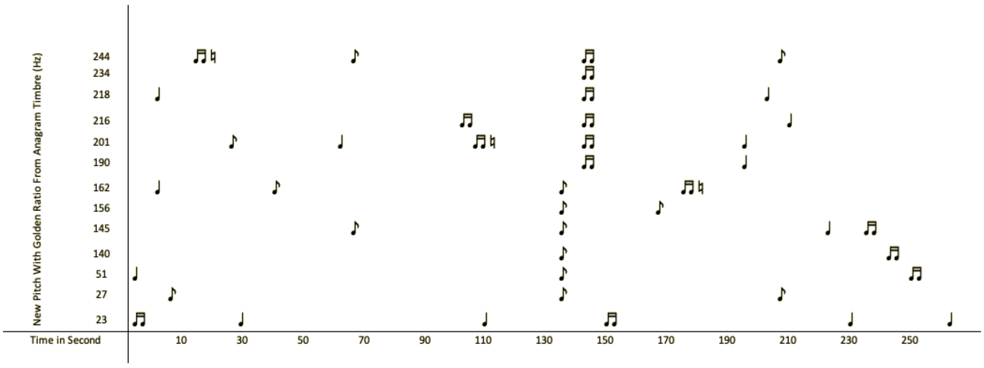

Figure 2. Music Notation “gold timbre” composition by Hery Budiawan.

Looking at this notation, researchers felt there was uniqueness in terms of new pitch and duration. Using a golden ratio of 1.618 obtained a time duration of 240 s and the highest pitch of 244Hz. This is quite surprising; with the use of the golden ratio Fibonacci, there is a balance of time and sound numbers. Figure 2 shows the musical notation of the composition, “gold timbre,” created by the researcher as a composer. This composition has a duration of 240 s, in which the generated pitch exceeds the range of sounds we commonly hear, ranging from 23 Hz to 244 Hz. To produce this work, computer media was employed since some sounds cannot be played using conventional musical instruments. The concept of “gold timbre” is built upon mathematical principles and the science of music composition. The music notation depicted in the image may encompass various musical signs and symbols that represent duration, pitch, dynamics, and the instruments used. The 240-second duration indicates the length of the composition and may contain interconnected sections with a smooth musical flow. It is important to note that the pitch produced in this composition lies outside the range of sounds we typically encounter in our daily lives. A pitch range of 23 Hz–244 Hz can create a unique and experimental sound character in this composition. This demonstrates that the composer creatively leveraged various musical elements to achieve the desired effects. The use of computer media in processing this work may be due to the difficulty or even impossibility of producing certain sounds or sound effects using conventional musical instruments. Computers provide flexibility and freedom for composers to create and manipulate sounds unhindered by the limitations of traditional musical instruments.

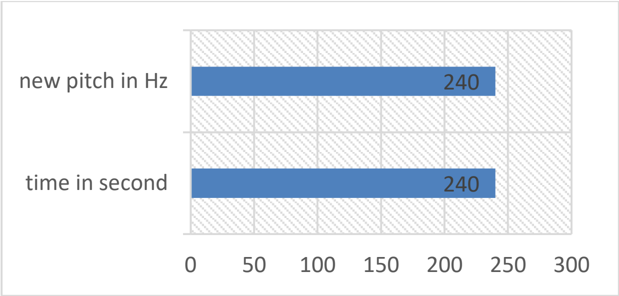

Figure 3. Time and pitch with golden ratio "gold timbre" music composition.

"Gold Timbre" is a concept that emerged through the integration of mathematics and the science of music composition. This concept may be based on principles such as the Fibonacci sequence, the golden ratio, or other musical principles that employ mathematical proportions and relationships to create compelling musical structures and characters. Overall, gold timbre exemplifies experimentation and innovation in the use of pitch beyond the usual range of sounds, the use of computer media for sound processing, and the application of interwoven mathematical and scientific concepts to music composition, reflecting creativity and artistic exploration in creating a unique and captivating musical composition.

4. Discussion

Integrating the Fibonacci sequence and timbre anagrams in the creation of a musical composition using the golden ratio concept opens up intriguing possibilities for composers. By dividing the Fibonacci sequence by the golden ratio, both note values and tempos can be derived. This approach allows for the creation of unconventional note values, which are approximated or rounded to more practical values for performance. This highlights the challenges of incorporating mathematical concepts into the realm of music (Shah, 2010; Viladot et al., 2018), where traditional instruments often rely on standard note values. Contemporary art music serves as a means to cultivate a connection with our experiential knowledge by reducing reliance on semantic interpretation and promoting a more direct and personal engagement with the auditory experience (Noble & McAdams, 2020; Wanke, 2023). However, the use of computer technology in processing these compositions enables the exploration of pitches that may not be feasible with traditional instruments, expanding the sonic possibilities for composers.

Some composers have also done this, contrasting their work with Fibonacci. Frédéric Chopin's Op. No. 28 Preludes, a collection of 24 remarkable musical miniatures, includes the genesis of the first prelude, inspired by a solitary game played by Chopin himself (Fisher, 2008)(Sacco, 2018). The composition, lasting about 30 s, is divided into two sections and features a climax at the 21st measure, corresponding to the golden ratio of the 34th measure. A similar placement of the climax is observed in Prelude Op. 28 No. 9 (BAKIM & YÖRE, 2020). While the preludes in Chopin's collection do not consistently adhere to the golden ratio, it is possible that Chopin did not view its use as necessary for musical success, as the preludes exhibit variations from this mathematical concept.

Figure 4. Prelude, No. 1 in C Major (Posamentier & Lehmann, 2007)(BAKIM & YÖRE, 2020)

The process of dividing timbre anagram pitches by the golden ratio also yields new pitches that can be incorporated into the composition. This presents an exciting opportunity to introduce different frequencies and tones, adding depth and richness to the overall musical experience. The varying differences between the original timbre anagram

pitches and the resulting new pitches provide a range of options for composers to explore and manipulate, highlighting the creative potential of the golden ratio in music composition (Mongoven, 2010) (Povilionienè, 2017). It also underscores the intricate relationship between mathematical principles and artistic expression (Franke, 1989)(Budiawan, 2023), highlighting how mathematical concepts can shape the composition's unique characteristics (An et al., 2008).
Table 6 summarizes the results obtained from applying the golden ratio concept as the divisor for the 13 Fibonacci sequences with timbre anagrams. These results serve as the foundation for creating a musical composition that embraces the Fibonacci golden ratio (Tatlow, 2006). The presented composition, processed using computer technology, demonstrates the innovative potential and new listening experiences that can arise from incorporating mathematical concepts into music. By pushing the boundaries of musical experimentation and leveraging mathematical principles, composers can venture into uncharted territories and captivate listeners with distinctive and harmonious compositions.
The use of computer-based tools in this creative process highlights the fusion of technology and artistic expression (Bevins, 2013; Branco & Zagalo, 2009; Cunningham et al., 2017b). While certain pitches derived from the golden ratio concept may be impractical to play with traditional instruments (BAKIM & YÖRE, 2020), the availability of computer technology allows composers to fully explore the potential of these pitches. This liberation from traditional constraints enables composers to venture into new sonic realms, expanding the possibilities for innovation in music creation (Menell, 2016; Netshivhambe, 2007) (Budiawan & Ananda, 2023) In contrast to Mozart's style, Mozart himself in his work also uses the Fibonacci golden ratio, which makes his work a balance both in terms of phrases and harmonies. One fascinating game played by renowned composers involves the intriguing use of the golden ratio to determine the structure of their works. Wolfgang Amadeus Mozart, known for his love of numbers and playful spirit, was particularly drawn to this practice. In his collection of 18 piano sonatas, all but one followed the sonata-allegro form, highlighting Mozart's fondness for this composition style (Noh, 2009). The remaining sonata, however, adopted the captivating "theme and variations" form. Upon closer examination, it is revealed that 6 of the 17 sonatas (35%) perfectly align with the golden ratio, as indicated by the word "golden" in the accuracy column of Table 2. Additionally, 8 of these works (47%) come remarkably close to the golden ratio, with error ratios ranging from −3 to +4, denoted in the accuracy column. These numbers signify the absence of a precise golden ratio. Notably, the remaining three works (18%), with high values of 6, 8, and 12, are not deemed closely associated with the golden ratio (Langston, 2020) (BAKIM & YÖRE, 2020) (Posamentier & Lehmann, 2007). The significance of the golden ratio in Mozart's compositions becomes evident when we analyze these examples from a statistical perspective, emphasizing its crucial role in shaping his musical masterpieces. Among the compositions listed in Table 2, Sonata No. 1 (K279) stands out as an exhibition of this profound influence. Marked as "golden," this composition comprises 100 measures, with its finale occurring precisely at the 38th measure. It can be argued that Sonata No. 1 exhibits the closest adherence to the golden ratio, further underscoring the impact of this mathematical concept on Mozart's musical craftsmanship. These findings shed light on the intentional implementation of the golden ratio in Mozart's work, indicating his meticulous attention to detail and the deliberate arrangement of musical elements. By strategically placing the climax or finale at specific measures aligned with the golden ratio, Mozart achieved a harmonious and aesthetically pleasing composition. Sonata No. 1 (K279) serves as a captivating example of how the golden ratio guided Mozart's artistic decisions, resulting in a symphony that resonates with mathematical beauty and musical brilliance.

Figure 5. Mozart's Piano Sonata, No.1 (K279) (Posamentier & Lehmann, 2007)

The correlation between Fibonacci numbers and music composition is profoundly evident in various aspects, notably in the arrangement of keys on a piano keyboard and the construction of Antonio Stradivarius's highly prized violins. The piano's keys, when examined within a single octave, reveal the manifestation of the first six Fibonacci sequence numbers. With an octave consisting of 13 keys (8 white and 5 black), the black keys are organized into sets of 2 and 3, showcasing a pattern aligned with the Fibonacci sequence (Garland, 1987).

Similarly, the meticulously studied proportions and components of Stradivarius's violins mirror the Fibonacci sequence numbers 2, 3, 5, 8, and 13 (Posamentier & Lehmann, 2007). In other respects, the principal objective of automatic melody harmonization is to construct a proficient model capable of generating a chord sequence. This generated chord sequence serves as a harmonically tailored accompaniment designed to complement a specified multibar melody sequence (Yin-Cheng Yeh Wen-Yi Hsiao & Yang, 2021). Delving further into the relationship between music and Fibonacci numbers necessitates an exploration of actual musical compositions. In Chopin's preludes, a notable recurring pattern emerges where the musical climax often coincides with the golden ratio division of the piece's length. Examining Prelude No. 1 in C major, spanning 34 measures, the climax at measure 21 closely approaches the golden ratio with a ratio of 34/21 = 1.619. A similar recurrence is observed in Prelude No. 9 in E major, where the climax at beat 29 nearly aligns with the golden ratio as 48/29 = 1.655. Although not universally present, this pattern reappears in various preludes, emphasizing the intentional integration of the golden ratio by composers (Posamentier & Lehmann, 2007) (Hartshorn, 2009) (Kumar et al., 2021). Beethoven's Fifth Symphony further exemplifies the golden ratio division in its first movement, where the opening five measures repeat 372 measures later and again after 228 measures, resulting in a ratio of 377/233 = 1.618 (Garland, 1987). Among Mozart's 17 piano sonatas employing the sonata-allegro form, six are precisely divided by the golden ratio, and eight are very close, indicating deliberate incorporation by Mozart (Posamentier & Lehmann, 2007) (Langston, 2020). The intentional use of the golden ratio extends beyond individual composers, with notable figures such as Haydn, Wagner, and Bartok, both from the classical and modern eras, intentionally incorporating this mathematical concept into their compositions (Posamentier & Lehmann, 2007) (Kumar et al., 2021). This deliberate use of the golden ratio in music stands out as particularly pronounced compared with its utilization in the realm of visual art. The integration of the Fibonacci sequence, timbre anagrams, and the golden ratio concept offers a unique approach to music composition. By leveraging these elements, composers can create compositions that incorporate unconventional note values and introduce new pitches. The interplay between mathematics and music opens up avenues of artistic expression that captivate listeners and offer fresh listening experiences. Through the use of computer technology, composers have the freedom to explore and push the boundaries of musical experimentation. This interdisciplinary approach highlights the potential for innovation and discovery when mathematical principles are integrated into artistic endeavors, highlighting the rich relationship between mathematics and music.

5. Conclusion

In conclusion, the profound influence of the Fibonacci sequence and the golden ratio on music composition, particularly in melody and harmony, is evident. Composers, whether experienced or novice, can create aesthetically pleasing and musically satisfying pieces by understanding the principles of the golden ratio. The application of the Fibonacci golden ratio, coupled with the use of timbre anagrams, plays a crucial role in the creation of the musical piece, where calculated values obtained through division serve as note values and tempos. The rounding or approximation of these values accommodates the challenges posed by unconventional note values in musical performance. Additionally, the exploration of new pitches through timbre anagrams adds a fresh dimension to the composition, enhancing its uniqueness. The range of differences between timbre anagram pitches and new pitches opens exciting possibilities for experimenting with various sonic elements. This integration of the golden ratio, Fibonacci sequence, and timbre anagrams highlights the potential constructive collaboration between mathematics and art, offering a unique sonic landscape for composers to explore. The study, exemplifying the researcher's commitment to pushing boundaries and embracing innovation, paves the way for future explorations in the realm of mathematical music composition, especially with the use of computer technology to realize pitches that may not be feasible with traditional instruments.

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